8. Algebraic Expessions and Operations on them

- A **variable** is something that does not have a fixed value. The value of a variable varies.
- Variables are represented by English letters such as x, y, z, a, b, c etc.
- A combination of variables, numbers and operators (+, –, × and ÷) is known as **expression**.

- Using different operations on variables and numbers, expressions such as $\frac{1}{7}-4y, 9x-5$, can be formed.

Example:

Meena's age is 4 years less than 7 times the age of Ravi. Express it using variables.

Solution:

Let the age of Ravi be x years.

7 times the age of Ravi can be expressed as $7x$.

4 years less than 7 times the age of Ravi can be written as $7x - 4$.

∴ Age of Meena = $(7x - 4)$

- Expressions can be classified on the basis of the number of terms present in them.
  - Expression containing only one term is called a **monomial**.

    For example, $2x$, $-3x^2$, $2xy$ etc.

  - Expression containing only two unlike terms is called a **binomial**.

    For example, $2x + 3$, $3x^2 - 2$, $-2xy + 3y^2$ etc.

  - Expression with three terms, where the terms are unlike is called a **trinomial**.

    For example, $2x^2 - 3x + 1$, $-2xy + 5y + 6x$ etc.

  In general, the expression with one or more terms is called a polynomial.

- **Degree** of the polynomial is the highest exponent of the variable in the polynomial.

  For example, polynomial $2x^2 - 3x + 1$ has degree 2.

- Addition and subtraction of linear algebraic expressions can be done using some algebraic properties and the concept of addition and subtraction of like terms.
- Some properties used in the addition and subtraction of algebraic expressions are:

$x - (y + z) = (x - y) - z$

$x - (y - z) = x - y + z$

For example, $(2a + 5b) - (a + b)$ can be simplified as follows:

$(2a + 5b) - (a + b) = (2a + 5b - a) - b$ $\{x - (y + z) = (x - y) - z$, where $x = (2a + 5b)$, $y = a$ and $z = b\}$

$= (2a - a + 5b) - b$

$= a + 5b - b$

$= a + 4b$

- The value of an algebraic expression depends upon the value of its variables.

  For example, the value of $(3a + 2b) - (2a + b)$ at $a = 1$ and $b = 3$ can be calculated as follows:

  $(3a + 2b) - (2a + b)$

  $= 3a + 2b - 2a - b$ $\{x - (y - z) = x - y + z$, where $x = (3a + 2b)$, $y = 2a$ and $z = b]$

  $= 3a - 2a + 2b - b$

  $= a + b$

  When $a = 1$ and $b = 3$ then the value of the given expression is:

  $1 + 3 = 4$

- The multiplication of a monomial by a monomial gives a monomial. While performing multiplication, the coefficients of the two monomials are multiplied and the powers of different variables in the two monomials are multiplied by using the rules of exponents and powers.

$(-2ab^2c) \times (3abc^2) = (-2 \times 3) \times (a \times a \times b^2 \times b \times c \times c^2) = -6a^2b^3c^3$

The multiplication of three or more monomials is also performed similarly.

$(xy) \times (3yz) \times (3x^2z^2)$

$= (3 \times 3) \times (x \times x^2) \times (y \times y) \times (z \times z^2)$

$= 9x^3y^2z^3$

- There are two ways of arrangement of multiplication while multiplying a monomial by a binomial or trinomial or polynomial. These are horizontal arrangement and vertical arrangement.

  Multiplication in **horizontal arrangement** can be performed as follows:

  Here, we arrange monomial and polynomial both horizontally and multiply every term in the polynomial by the monomial by making use of distributive law.

  $5a \times (2b + a - 3b + c)$

  $= (5a \times 2b) + (5a \times a) + (5a \times (-3b)) + (5a \times c)$

  $= 10ab + 5a^2 - 15ab + 5ac$

  $= 5a^2 - 5ab + 5ac$

  Multiplication in **vertical arrangement** can be performed as follows:

$$\begin{array}{r} 4x^2 + 2x \\ \times \quad 3x \\ \hline 12x^3 + 6x \\ \hline \end{array}$$

Here, we have first multiplied $3x$ with $2x$ and wrote the product with sign at the bottom. After doing this, we have multiplied $3x$ with $4x^2$ and wrote the product with sign at the bottom.

Similarly, we can multiply a trinomial with monomial as follows:

$$\begin{array}{r} 2y^3 - 5y + 1 \\ \times \qquad 2y \\ \hline 4y^4 - 10y^2 + 2y \\ \hline \end{array}$$

- While multiplying a polynomial by a binomial (or trinomial) in horizontal arrangement, we multiply it term by term. That is, every term of the polynomial is multiplied by every term of the binomial (or trinomial).

  Example:

  Simplify $(x + 2y)(x + 3) - (2x + 1)(y + x + 1)$.

  Solution:

  $(x + 2y)(x + 3) = x(x + 3) + 2y(x + 3)$

  $= x^2 + 3x + 2xy + 6y$

  $(2x + 1)(y + x + 1) = 2x(y + x + 1) + 1(y + x + 1)$

  $= 2xy + 2x^2 + 2x + y + x + 1$

  $= 2xy + 2x^2 + 3x + y + 1$

  $\therefore (x + 2y)(x + 3) - (2x + 1)(y + x + 1) = x^2 + 3x + 2xy + 6y - 2xy - 2x^2 - 3x - y - 1$

  $= -x^2 + 5y - 1$

- We can also perform multiplication of two polynomials using vertical arrangement.

  For example,

$$\begin{array}{r} l + 6m + 7n \\ \times \qquad l + 3m \\ \hline 3lm + 18m^2 + 21mn \\ +l^2 + 6lm + 7nl \\ \hline l^2 + 9lm + 18m^2 + 21mn + 7nl \\ \hline \end{array}$$

- **Solving an equation by performing same mathematical operation on both sides:**

It is known that an equation remains unchanged on adding or subtracting the same number on both sides. Therefore, using this property, an equation can be solved.

Consider $3x - 7 = 2$

Adding 7 on both sides, we obtain

$3x - 7 + 7 = 2 + 7$

$\Rightarrow 3x = 9$

Dividing both sides by 3, we obtain

$\frac{3x}{3} = \frac{9}{3}$

$\Rightarrow x = 3$

Therefore, $x = 3$ is the solution of $3x - 7 = 2$